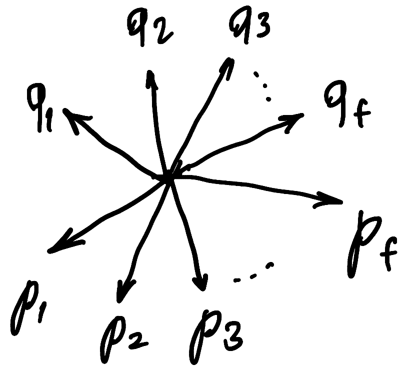
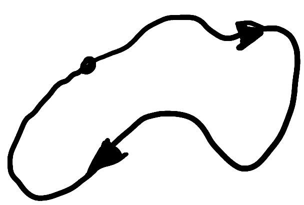
Phase space. Distribution function.

<u>Phase space</u> - space consisting of coordinates and momenta



Because Hamilton's equations are 1-st-order differential equations, a point in phase space specifies uniquely all the subsequent evolution



Phase trajectory

As an <u>example</u>, consider a harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

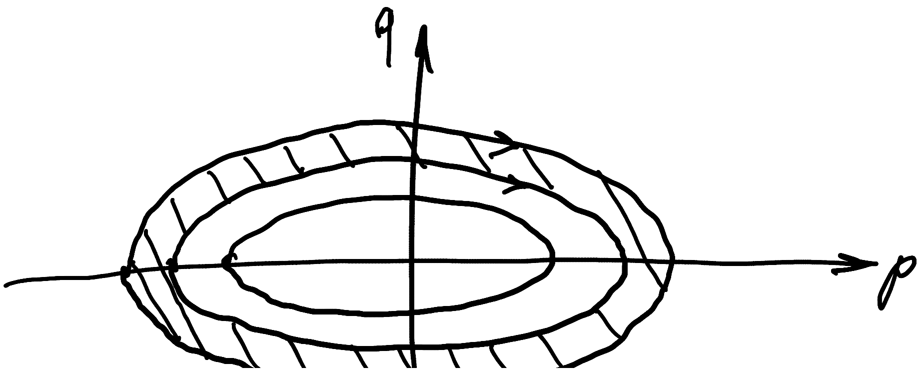
$$\begin{cases} \dot{p} = -m\omega^2 q \\ \dot{q} = \frac{p}{m} \end{cases} \longrightarrow \ddot{q} + \omega^2 q = 0$$

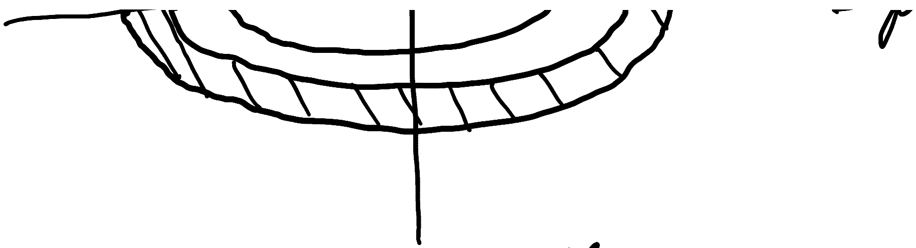
$$\downarrow$$

$$q = A \sin(\omega t + \varphi)$$

$$p = A m\omega \cos(\omega t + \varphi)$$

$$\left(\frac{q}{A}\right)^2 + \left(\frac{p}{m\omega A}\right)^2 = 1$$ - equation of motion in phase space





The area of the ellipse $S = \pi m \omega A^2$

Energy $\varepsilon = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} = \frac{1}{2} A^2 m\omega^2 \cos^2 + \frac{1}{2} A^2 m\omega^2 \sin^2 = \frac{1}{2} A^2 m \omega^2$

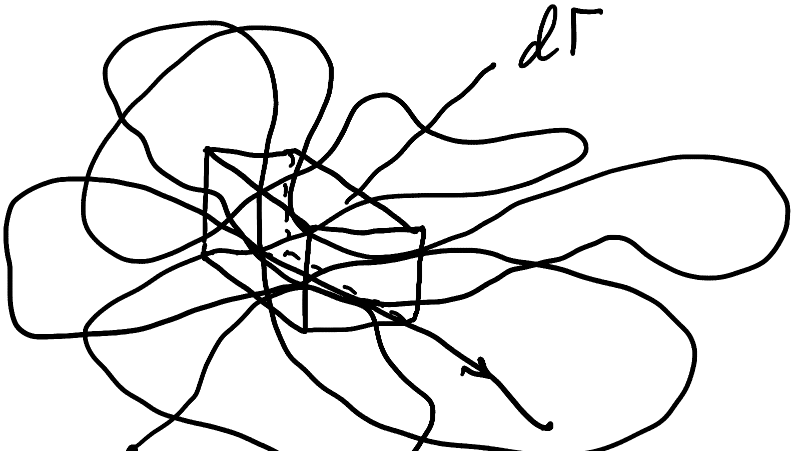
$S = \oint p\, dq = 2\pi \frac{\varepsilon}{\omega} = 2\pi \hbar N$

Transition to quantum case

The concept of phase space is still very usefull for a system of many particles

For N particles in 3D

$$d\Gamma = \underbrace{dq_1\, dq_2 \ldots dq_{3N}}_{dq \text{ Infinitesimal el-nt in the configuration space}}\ \underbrace{dp_1\, dp_2 \ldots dp_{3N}}_{\text{—''— phase space } dp}$$



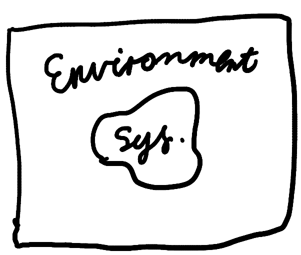
The statistical approach is based on the assumption that the system will go through a part of phase space many times assumption that the system ... given element of phase space many times

$$dw = \lim_{T \to \infty} \frac{dt}{T}$$ - prob-ty to find the system there

$$dw = \underbrace{\rho(p_1, p_2, \dots, p_f ; q_1, q_2, \dots, q_f)}_{\text{statistical distribution function}} \, dp \, dq$$

$$\int \rho \, dp \, dq = 1$$

In an equilibrium system the distribution function is independent of the initial conditions. That applies to a system in an external environment as well.



That problem is way easier than solving the microscopic equations of motion

The probability of quantity f:

$$\bar{f} = \int f(p,q) \, \rho(p,q) \, dp \, dq$$

If there are 2 independent systems

$$\rho_{12} \, dp^{(12)} dq^{(12)} = \rho_1 \, dp^{(1)} dq^{(1)} \cdot \rho_2 \, dp^{(2)} dq^{(2)}$$

$$\rho_{12} = \rho_1 \rho_2$$

$$\langle f_1 \; f_2 \rangle = \langle f_1 \rangle \langle f_2 \rangle$$